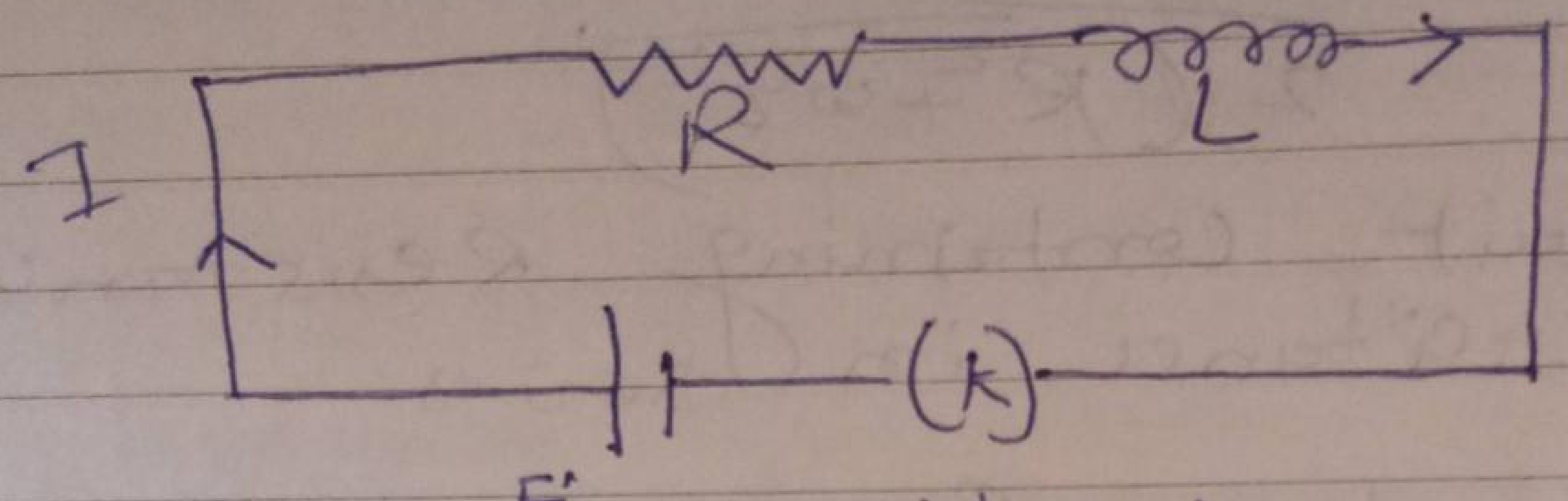
Que

Growth and Decay of current in a circuit containg inductance and Resistance



circuit diagram

let us consider a circuit containing a coil of self Inductance L and a wire of Resistance R and a cell of constant e.m.f E connected in series. when the circuit is closed by throwing the switch (k) a self-induced e.m.f is set up in the coil which opposes the growth of current in the circuit. Hence the current does not reach its final steady value E/R instantaneously.

During the variable state when the current is growing, I current at any instant t

$\therefore$ opposing induced e.m.f $= L\,dI/dt$

$\therefore$ Effecting e.m.f $= E - L\,dI/dt$

According to ohm's law, the effective e.m.f of the circuit will be equal to the potential drop across the resistance $R = RI$

$\therefore E - L\,dI/dt = RI$

or $E - RI = L\,dI/dt$

$$dI/dt = \frac{E - RI}{L}$$

$$\frac{L\,dI}{E - RI} = dt$$

integrating it

$$-\frac{L}{R}\log_e(E - RI) + C = t$$

where C is constant of integrating

$t = 0 \quad I = 0$

$$C = L/R \log_e E$$

or $t = -\frac{L}{R}\log_e(E - RI) + L/R \log_e(E)$

or $\frac{R}{L}t = \log_e\frac{(E - RI)}{E} = \log_e(1 - \frac{RI}{E})$

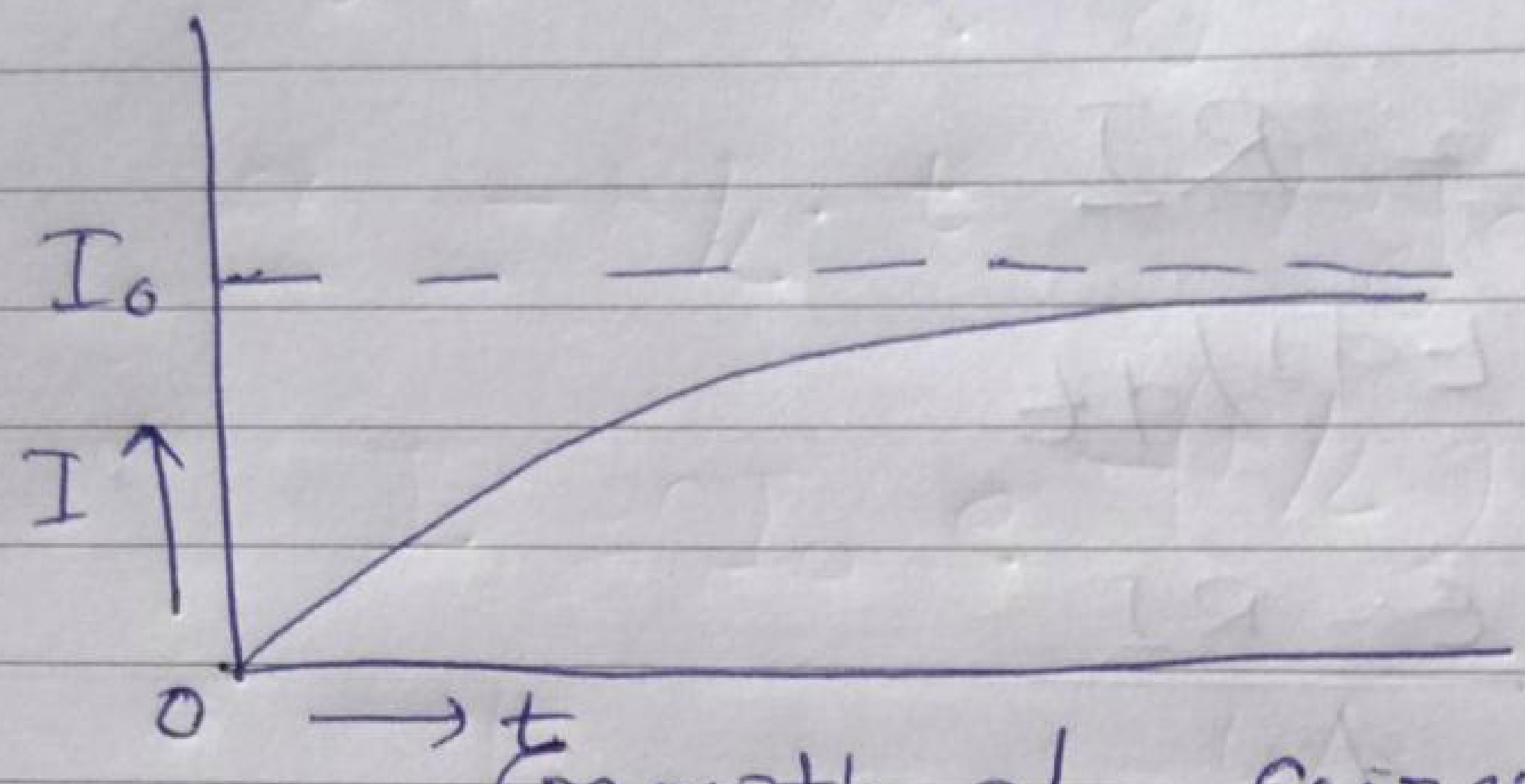
or $e^{-R/L\,t} = 1 - \frac{RI}{E}$

or $I = E/R\,[1 - e^{-(R/L)\cdot t}]$

where $E/R = I_0$ Max^m current

$$\boxed{I = I_0[1 - e^{-(R/L)t}]}$$

This equation shows that the current in the circuit rises according to an exponential law.



Growth of current

$$dI/dt = -I_0\left(-\frac{R}{L}\right) e^{-(R/L)t}$$

$$= I_0 R/L \, e^{-(R/L)t}$$

$$I = I_0(1 - e^{-R/L\,t})$$

$$\frac{I}{I_0} = 1 - e^{-R/L\,t}$$

$$1 - I/I_0 = e^{-R/L\,t} = \frac{I_0 - I}{I_0}$$

$$\boxed{dI/dt = \frac{R}{L}(I_0 - I)}$$

The rate of growth of current depends only on the ratio R/L which is called the time constant of the circuit

$$L/R = \lambda$$

$$I = I_0 [1 - e^{-t/\lambda})$$

at $t = \lambda$

$$I = I_0 (1 - 1/e) = I_0 \left(\frac{e-1}{e} \right) = I_0 \left(\frac{2.7187}{2.718} \right)$$

$$= I_0 [0.632]$$

Thus $(t = \lambda)$ time constant of the circuit is the time in which the current attains 0.632 of its maximum steady value.

Decay of current → when key K is open the e.m.f E applied to the circuit becomes zero. The resistance of the circuit remains unchanged. The self inductance of the coil opposes the fall of the current. During the variable state when the current is ~~falling~~. let I be the current at any instant t. There is no applied e.m.f in the circuit, the ~~potential~~ potential drop RI across the resistance R is equal and opposite to the induced e.m.f. $L dI/dt$

$$\therefore -L\, dI/dt = RI$$

$$dt = -L/R \; dI/I$$

Integrating it

$$\text{or} \int dt = -L/R \int dI/dt$$

$$\therefore t = -L/R \log_e I + C$$

where C is integration constant

$$\text{At } t = 0 \quad I = I_0$$

$$\therefore C = L/R \log_e I_0$$

putting this value in eqn

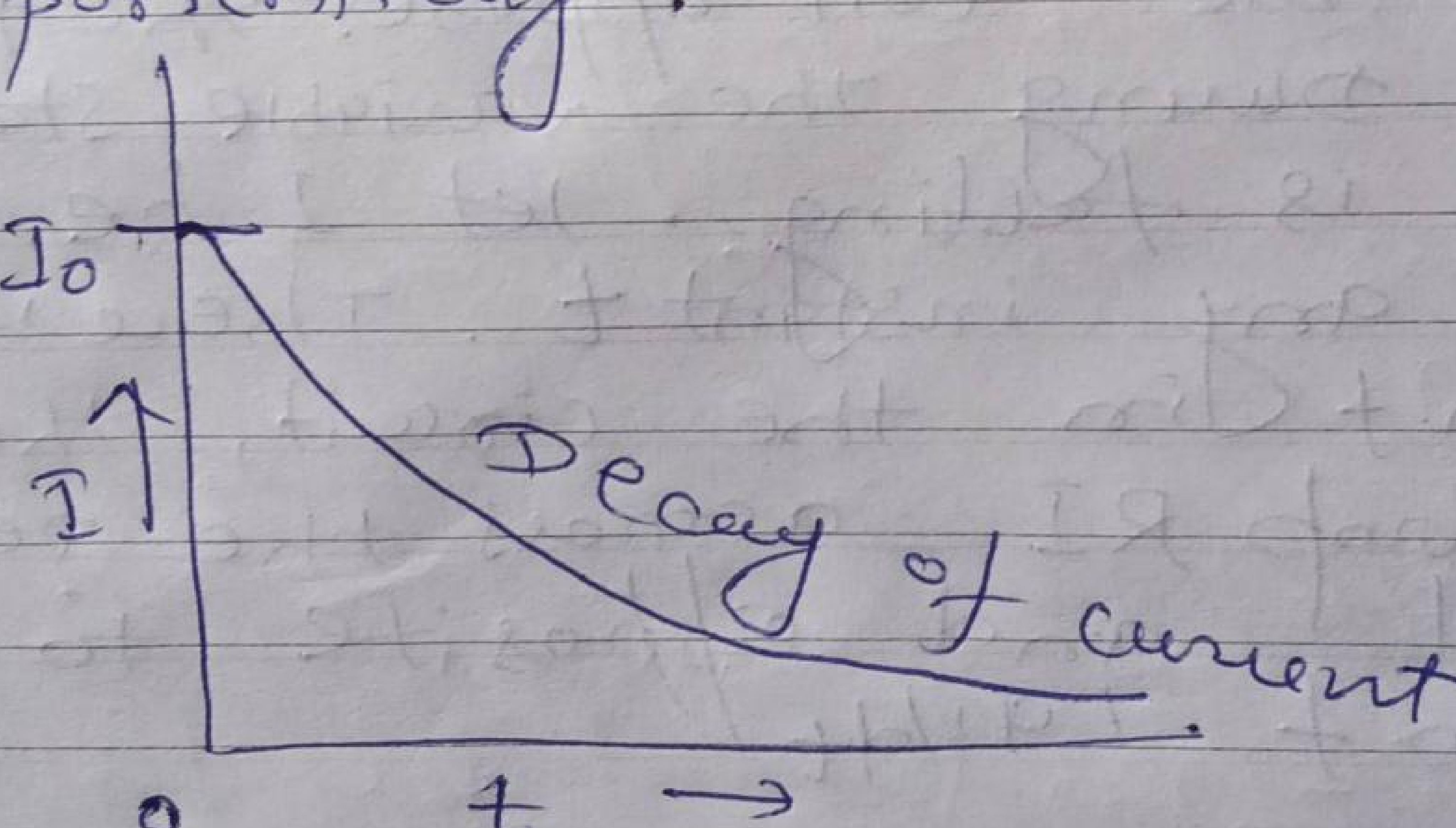
$$t = -L/R \log_e I + L/R \log_e I_0$$

or $$-R/L \cdot t = \log_e I - \log_e I_0$$

$$I = I_0 e^{-(R/L)t} = I_0 e^{-t/\lambda}$$

$$\boxed{I = I_0 e^{-t/\lambda}}$$

This equation shows that the decay in current in the circuit is exponentially.



$$dI/dt = -I_0 R/L\, e^{-(R/L)t} = I_0 \frac{R}{L} e^{-t/\lambda}$$

$$dI/dt = -\cancel{I_0} R/L \cdot I/\cancel{I_0} = -\frac{R}{L} I$$

$\lambda = L/R$ = time const

$$\frac{I}{I_0} = 1/e = 37\%.$$